The Compliant Capacitor

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Most materials development has been driven by human needs

- Shelter, defense, transportation, entertainment ..... 
- Strong, stiff, unyielding, hard ..... 

Some materials have been developed for replacing structural body parts

- Artificial teeth, hip implants.....

Only now are materials being developed that are compatible with humans and extending their capabilities

- Soft, compliant, conformable, capable of a range of motion.......

NB. Strictly, skin is a viscoelastic material not elastic so it has no true elastic modulus. But one can speak about instantaneous modulus.
We use muscles for

• walking and running
• dancing
• swimming
• getting up and sitting down
• moving our eyes
• swinging our arms
• turning our head
• gripping, holding and letting go
• ......
## Approximate Range of Characteristics of Existing Mechanical Actuators

<table>
<thead>
<tr>
<th>actuator type</th>
<th>$\varepsilon_{\text{max}}$ [-]</th>
<th>$\sigma_{\text{max}}$ (MPa)</th>
<th>$E$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>low strain piezoelectric</td>
<td>$5 \times 10^{-6} - 3 \times 10^{-5}$</td>
<td>1–3</td>
<td>90–300</td>
</tr>
<tr>
<td>high strain piezoelectric</td>
<td>$5 \times 10^{-5} - 2 \times 10^{-4}$</td>
<td>4–9</td>
<td>50–80</td>
</tr>
<tr>
<td>piezoelectric polymer</td>
<td>$2 \times 10^{-4} - 1 \times 10^{-3}$</td>
<td>0.5–5</td>
<td>2–10</td>
</tr>
<tr>
<td>thermal expansion (10 K)</td>
<td>$9 \times 10^{-5} - 3 \times 10^{-4}$</td>
<td>20–50</td>
<td>70–300</td>
</tr>
<tr>
<td>thermal expansion (100 K)</td>
<td>$9 \times 10^{-4} - 3 \times 10^{-3}$</td>
<td>200–500</td>
<td>70–300</td>
</tr>
<tr>
<td>magnetostrictor</td>
<td>$6 \times 10^{-4} - 2 \times 10^{-3}$</td>
<td>90–200</td>
<td>40–200</td>
</tr>
<tr>
<td>shape memory alloy</td>
<td>$7 \times 10^{-3} - 7 \times 10^{-2}$</td>
<td>100–700</td>
<td>30–90</td>
</tr>
<tr>
<td>moving coil transducer</td>
<td>$1 \times 10^{-2} - 1 \times 10^{-1}$</td>
<td>$4 \times 10^{-3} - 5 \times 10^{-2}$</td>
<td>$4 \times 10^{-5} - 5 \times 10^{-3}$</td>
</tr>
<tr>
<td>solenoid</td>
<td>$1 \times 10^{-1} - 4 \times 10^{-1}$</td>
<td>$4 \times 10^{-2} - 1 \times 10^{-1}$</td>
<td>$3 \times 10^{-4} - 1 \times 10^{-3}$</td>
</tr>
<tr>
<td>muscle</td>
<td>$3 \times 10^{-1} - 7 \times 10^{-1}$</td>
<td>0.1–0.4</td>
<td>$5 \times 10^{-3} - 2 \times 10^{-2}$</td>
</tr>
<tr>
<td>pneumatic</td>
<td>$1 \times 10^{-1} - 1 \times 10^{0}$</td>
<td>0.5–0.9</td>
<td>$5 \times 10^{-4} - 9 \times 10^{-4}$</td>
</tr>
<tr>
<td>hydraulic</td>
<td>$1 \times 10^{-1} - 1 \times 10^{0}$</td>
<td>20–70</td>
<td>2–3</td>
</tr>
</tbody>
</table>

Capacitors

What they do:
These store and release electrical energy. They are widely used to absorb electronic noise and detect signals of set frequencies.

Size: 0.4mm x 0.2mm to 1mm x 0.5mm
Number per phone: 700

Notes: Numbers per smartphone are approximate. Source: Murata Manufacturing
A capacitor is usually used to store electrical energy so it requires a large dielectric constant and large area. Electrical energy stored is:

\[ U_{\text{electrical}} = \frac{\varepsilon \varepsilon_0 V^2 A h}{2 h^2} \]

Capacitance:

\[ C = \frac{Q}{V} = \frac{\sigma_s A}{E h} = \varepsilon \varepsilon_0 \frac{A}{h} \]

Attractive force between opposite charges is:

\[ P_c = \frac{1}{2} \varepsilon \varepsilon_0 E^2 \]

induces electrostrictive strain that for most dielectrics is ppm, ie nanometers
Freeing the Capacitor -- The Compliant Capacitor

Applying a voltage to a capacitor produces opposite charges in the electrodes across the dielectric.

Coulombic attraction between the charges creates a Maxwell force that compresses the dielectric (equivalent to electrostatic actuator in MEMS)

\[ P = \sigma_z = \varepsilon \varepsilon_o E^2 = \varepsilon \varepsilon_o \left( \frac{V}{h} \right)^2 \]

produces strain \( s_z = -\frac{\sigma_z}{Y} \)

Elastomers are incompressible, ie volume change = 0. So, if reduce thickness, expands laterally

\[ \Delta V = 0 = s_x + s_y + s_z : \quad s_x = s_y = \frac{P}{2Y} = \varepsilon \varepsilon_o \frac{V^2}{2Y h^2} \]
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Actuation stretch

\[ \lambda \approx \frac{P}{2Y} \approx \varepsilon \varepsilon_o \frac{V^2}{2Y h^2} \]

\[ \text{Voltage} \]

\[ \text{Elastomers are incompressible, ie volume change = 0. So, expands laterally} \]

\[ \text{Replacing a ceramic dielectric with an elastomer leads to a million times greater strain!} \]
During actuation, the voltage increased at a rate of 20 V/s until electrical breakdown.

VHB 4905 elastomer (Acrylic-based)  
Prestretched to 250%

Quantify actuator deformation, stretch, $\lambda$

$$\lambda = \frac{r}{r_o}$$

As elastomers are incompressible, so

$$\lambda^2 = \frac{H}{h}$$

Carbon nanotube electrodes

During actuation, the voltage increased at a rate of 20 V/s until electrical breakdown.

$$\lambda = \frac{r}{r_o} \approx 2$$
• Loosely cross-linked, long chain molecules
• Transparent materials
• Highly insulating
• Dielectric constant ~ 2.6
• On stretching – chains straighten
• Huge, nonlinear strain capabilities
• Incompressible – Poisson ratio 0.5

Typical elastomers:
• Natural rubber (latex)
• PDMS
• Arcylic (3M VHB 7905)
• Polyurethane
Compliant capacitor as an electrically driven artificial muscle
A Conventional Electrically Focusing Camera

Focal length is constant

Focusing by moving lens

Limitations of traditional lenses:

- Bulky
- Slow
- Complicated design, multiple gears
- Noisy
- Limited focusing range

\[
\frac{1}{f} = \frac{1}{S_1} + \frac{1}{S_2}
\]
The Human Eye: Focusing

Focusing is associated with change in shape of the ocular lens

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An alternative approach:

*Change focal length of the lens by changing its curvature*

Solution:

Constant volume liquid droplet contained between transparent sheets whose areas can be altered with an applied voltage.

Requires:

Transparent, compliant electrodes
Demonstration of a Tunable Lens

Object Distance

16 cm  32 cm  100 cm  770 cm

Focal length change, $\Delta f/f_{0}$ (%)

$D_1/D_2 = 1.6$

Initial focal length $f_{0} = 36$ mm

Actuation voltage, V
High Speed Response

0 V and 4.5 kV
The scale is in mm.

(a) 5 Hz 10 Hz
(b) 4500 V

0 V
90%
25 ms
10 ms
90%
Calculating the dynamic range of focal length:
- Constant volume
- Spherical curvature, h/a < 0.25
- DEA membrane, D2 → curvature decrease
- Passive membrane, D1 → curvature increase

\[ \Delta f_{max} = \frac{f_{max} - f_o}{f_o} \times 100\% \]

Color indicate maximum focal change
The maximum focal length variation ($\Delta f_{\text{max}}$) depend on relative diameter of the membranes ($D_1/D_2$), but independent of the refractive index.
The Compliant Capacitor
as a Machine Element
Machine Elements of Soft Robotics

- Actuators
  - Linear actuators
  - Biaxial actuators
- Rotary drives
- Soft clamps
- .............
Devising A Uniaxial Actuator

Complaint capacitor creates a biaxial strain.

How do we use it to create uniaxial strain?

Apply through thickness electric voltage.
Equi-biaxial strain

Insert aligned and parallel stiffeners
Apply electric voltage
Elongation perpendicular to stiffeners
Uni-axial Fiber Stiffening of Dielectric Elastomer

Cylindrical geometry chosen to minimize electrical breakdown at edges
Uni-axial Actuator with no Stiffening
Cylindrical Actuator Using Fiber Stiffened Elastomer

(a)

(b)

(c)

(d)

Voltage, $\phi$ (kV)

Actuation Strain, $\xi_2^{\text{act}}$

$\Delta L=4.0\text{cm}$

$\Delta L=10.0\text{cm}$
Uniaxial Actuator
Mimicking Finger Action

Fingers gripping an object

Fingers striking piano keys

Mechanical element analog: unimorph or bimetallic strip
The Simplest Robotics Element: The Unimorph

- End deflection
- Blocking force

(a) Unimorph DEA
(b) Dielectric Elastomer
(c) Substrate

Diagram showing the configuration of the unimorph and its components.
Dielectric Elastomer Unimorph Actuation

NB. Transparent electrodes

Curling instability due to field induced curvature in two mutually perpendicular directions
The Dielectric Elastomer Based Unimorph

Expansion parallel and perpendicular to beam axis, causes bending in two perpendicular directions. As the strains can be large, the out-of-plane bending can be large.
Unimorph Actuation

Observations

Finite element computations
Aligned fibers produce higher stiffness parallel to fibers than perpendicular to them. $E_{\text{nylon}} \gg E_{\text{VHB}}$. This breaks the flexural deformation response symmetry of the beam.
Simulations Show Suppression of Curling
Simulations Show Suppression of Curling

A single fiber can be sufficient to break deformation symmetry
Fiber Stiffened Dielectric Elastomer Based Unimorph

Fabrication:

(a) CNT electrode mat
(b) VHB, 0.25mm
(c) Nylon Fibers
(d) VHB, 0.25mm
(e) CNT electrode mat
(f) VHB, 0.5mm

Assembled actuator

Optical Micrograph:
An Inchworm Based on Fiber Stiffened Unimorph
Effect of Elastic Anisotropy in Unimorphs

Low volume fraction of aligned fibers create elastic anisotropy and alters response.
A Simple Soft Robotics Component: The Gripper
Mechanical Energy Harvesting
with a Compliant Capacitor
## Energy Harvesting Schemes

<table>
<thead>
<tr>
<th>Energy Source</th>
<th>Characteristics</th>
<th>Harvested Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>Outdoor</td>
<td>100 mW/cm²</td>
</tr>
<tr>
<td></td>
<td>Indoor</td>
<td>100 μW/cm²</td>
</tr>
<tr>
<td>Thermal</td>
<td>Human</td>
<td>60 μW/cm²</td>
</tr>
<tr>
<td></td>
<td>Industrial</td>
<td>~1-10 mW/cm²</td>
</tr>
<tr>
<td>Vibration</td>
<td>~Hz—human</td>
<td>~4 μW/cm³</td>
</tr>
<tr>
<td></td>
<td>~kHz—machines</td>
<td>~800 μW/cm³</td>
</tr>
<tr>
<td>RF</td>
<td>GSM 900 MHz</td>
<td>0.1 μW/cm²</td>
</tr>
<tr>
<td></td>
<td>WiFi</td>
<td>0.001 μW/cm²</td>
</tr>
</tbody>
</table>

### Examples of Energy Harvesting Devices:

- **Watch**: ~5μW
- **Smoke detector**: 6μW
- **Occupancy motion detector**: 28μW
- **LCD clock**: ~500μW
- **Glass breakage**: 1.9mW-32mW
- **Seismic sensor**: 37mW
- **Headphones**: ~60mW
- **Smartphone**: ~1W

**Power Ranges:**

- 1 μW
- 10μW
- 100μW
- 1mW
- 10mW
- 100mW
- 1W+
Small-Scale Energy Harvesting

Fig. 2. Shrinking IC chip line geometries and lower power consumption levels come at a time when energy-harvesting devices are becoming more effective and practical. (Source: IDTechEx).
Harvesting Electrical Energy Through Mechanical Work

Convert mechanical energy to elastic energy

Convert elastic energy to electric energy through change in capacitance

Change in stored elastic energy:
\[ \Delta E_M = \int P \, dL \]

Recall: Electrical energy of a capacitor is:
\[ E_{\text{electrical}} = \frac{\varepsilon \varepsilon_0 V^2}{2 h^2} = \frac{C V^2}{2} \]

Essentially a device for transferring charge from a low to high voltage – A voltage Step-Up Transformer
What is Optimum Mechanical Loading Configuration?

- Reference State
- Uniaxial Stretch
- Pure Shear Stretch
- Equi-biaxial Stretch

Capacitance Change:

- Reference State: $C \propto \frac{\lambda}{(1/\sqrt{\lambda})} = \lambda$
- Uniaxial Stretch: $C \propto \frac{\lambda}{(1/\lambda)} = \lambda^2$
- Pure Shear Stretch: $C \propto \frac{\lambda\lambda}{(1/\lambda^2)} = \lambda^4$
Harvesting Mechanical Energy: Energy Cycle

Energy density: \[ E_{\text{Density}} = \frac{\Delta E_{\text{Net}}}{M} \]
In our experiments, $\lambda$ changes in the range from 1.2 to 5.4.
Movie

Top down View:

Side View:
Measured Energy Harvesting Cycles

Schematic

Voltage, $\Phi_{DE}$ vs. Stretch, $\lambda$

Voltage, $\Phi_{DE}$ vs. Charge on DE, $Q_{DE}$ (μC)

0.5 Hz cycles
Verification of Stretch Scaling

Model indicates that capacitance scales with fourth-power of stretch under biaxial loading

\[ C \propto \lambda^4 \]

Max stretch is 4.25
Representative Generator Cycles

Electrical Energy from Power Source:

\[ \Delta E_{in} = \Phi_L \Delta Q_{in} = \Phi_L \int_{T_0}^{T+T_0} i_{\text{charge}} \, dt \]

Electrical Energy Harvested:

\[ \Delta E_h = \Phi_H \Delta Q_h = \Phi_H \int_{T_0}^{T+T_0} i_{\text{harvest}} \, dt \]
The average energy density of the first eight cycles is $560\text{J/kg}$ with a power density of $280\text{W/kg}$. 

$\Delta E_{\text{Net}} = \Delta E_h - \Delta E_{\text{in}}$

$E_{\text{Density}} = \frac{\Delta E_{\text{Net}}}{M}$
Improved Harvesting Cycle?

Shian et al., *Adv. Mater.* 2014,
DOI: 10.1002/adma.201402291
NB. Mechanical energy storage is separated from the subsequent energy conversion so can occur non-uniformly or even intermittently while energy conversion can occur over a shorter time.
Improved Harvesting Cycle

Complex Shapes From Bi-strips:
Pre-straining, Joining and Release Operation

1. Two elastomer strips


4. Release stretching force

2. Pre-strain

Pre-strain :
\[ \chi = \frac{(L - L')}{L'} \]
Release of a narrow bi-strip to form a Hemi-helix

Perversions

After release

After release and rotating one end

---- a regular helix
Examples of Morphological Shape Transitions
Elastomers Offer Opportunities For New Machines and Devices

- Elastomers are soft, compliant, stretchable
- Elastically compatible with humans
- Ideal for Soft Robotics
- Shape changes can be electrically actuated
- Large scale elastic effects
- Transparent but can be colored
- Combination ideal for new optic and mechanical devices
- Elastomer sheets are mass produced